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The Return of the Sampling Frequency Convertor

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ABSTRACT

This paper explains how sampling frequency convertors can be used to acquire broadband modulated waveforms. Their use is illustrated with a practical measurement example performed on a "Large-Signal Network Analyzer".

INTRODUCTION

Frequency convertors are an important part of large-signal network analyzers. They convert signals which have a frequency in the GHz range to a much lower frequency, typically in the MHz range. This frequency conversion to a much lower frequency is needed in order to be compatible with the bandwidth restrictions of existing analog-to-digital convertors. Two types of frequency convertors exist: mixing frequency convertors and sampling frequency convertors. In general, mixer based convertors have the best sensitivity and noise rejection characteristics, and sampler based convertors have the simplest architecture and the highest "instantaneous" bandwidth (by this is meant the bandwidth covered by one acquisition cycle of the analog-to-digital convertor). In this paper it is shown that a sampling frequency convertor, unlike a mixing frequency convertor, can measure signals which have a modulation bandwidth which is many times higher than the bandwidth of the analog-to-digital convertor that is used. This makes the sampling frequency convertor ideal for advanced large-signal network analyzer applications.

SAMPLING FREQUENCY CONVERTOR BASICS

The schematic of a multichannel sampling frequency convertor is depicted in Fig. 1. The microwave input signals x_{RF}, y_{RF},... enter through matched transmission lines and travel towards the sampling nodes of each channel. At the sampling node there is a switch which repetitively closes for a very short time. The duration of the closing of the switch, which is called the sampler aperture, is typically 10 ps. The rate at which the switch closes and opens is called the sampling frequency, it is noted f_S. The sampling frequency is generated by a repetitive pulse generator which has a very fine frequency resolution. A typical value for the range of f_S is 10 MHz to 20 MHz, with a resolution as fine as 1 mHz. Each time the switch closes it captures a small amount of electrical charge from the transmission line center conductor, where this amount of charge is proportional to the average voltage at the sampling node during the sampler aperture. The sampled charge is filtered by a low pass filter having a cut-off frequency f_{LP} which results in the frequency converted signals x_{IF}, y_{IF},... The mathematical theory, as well as more details on the hardware, can be found in [1]. For now, we will assume that

$$f_S > 2f_{LP} . \qquad (1)$$

The condition described by (1) is valid in practice for the LSNA prototypes as developed by the Hewlett-Packard Company and Agilent Technologies. Under the condition described by (1) the frequency conversion characteristics as described in [1] can be summarized as follows. Assume that the convertor input signal x_{RF} is a sinusoidal signal with a frequency f_{RF} which is represented by the phasor A_{RF}. One also assumes that the transmission lines and the low pass filter in Fig. 1

introduce negligible distortions. The output signal x_{IF} will then be a sinusoidal signal with a frequency f_{IF} which is represented by a phasor A_{IF}. A_{IF} and f_{IF} are given by equations (2), (3) or (4), depending on the case.

if $(f_{RF} \bmod f_S) < f_{LP}$ then $A_{IF} = A_{RF}$ and $f_{IF} = (f_{RF} \bmod f_S)$ (2)

if $(-f_{RF} \bmod f_S) < f_{LP}$ then $A_{IF} = \text{conj}(A_{RF})$ and $f_{IF} = (-f_{RF} \bmod f_S)$ (3)

if $(f_{RF} \bmod f_S) > f_{LP}$ and $(-f_{RF} \bmod f_S) > f_{LP}$ then $A_{IF} = 0$ (4)

In the above equations the notation conj(.) stands for complex conjugate and (x mod y) stands for the modulo operator meaning the "remainder of x divided by y". The modulo operator is defined for both negative and positive quantities for x, where the following relationship holds:

$$(-x \bmod y) = y - (x \bmod y). \quad (5)$$

Note that in general $(-x \bmod y) \neq -(x \bmod y)$, which is counter-intuitive to many people.

As readily can be verified the condition described by (1) ensures that at most one frequency component will be present in x_{IF}. Expressed in words, these equations indicate that there are three possibilities for x_{IF}: a direct harmonic mixing product described by (2), an image harmonic mixing product described by (3) or no detectable harmonic mixing product described by (4). The latter is the case when all mixing products are removed by the low pass filter. Note the conjugate operator in (3) which describes the case of the image mixing product.

In a practical measurement x_{RF} may contain many different tones. In this case the frequency conversion as described in (2) to (4) will apply to each tone separately.

As is evident from (2) and (3) the frequency conversion is mathematically described by a modulo operator, where the second argument is f_S. In the following paragraphs will be shown how a smart choice of the signal frequencies and the sampling frequency results in unique modulation measurement capability which is unattainable by mixing frequency convertors.

CONVERSION OF A FUNDAMENTAL CARRIER AND HARMONICS

As a first simple case one considers the measurement of a microwave carrier with N harmonics as described in [1]. In this case x_{RF} is a multitone signal described by N phasors A_{RFk} with frequency f_{RFk}, with k being the harmonic index. This is described in (6).

$$f_{RFk} = k\, f_C \quad (6)$$

The set of frequencies f_k for k ranging from 0 to N is called the signal "frequency grid". In order to perform the frequency conversion the value of f_S is chosen such that

$$(f_C \bmod f_S) < \frac{f_{LP}}{N}. \quad (7)$$

It can readily be shown that there is always a solution for f_S within the frequency range of the repetitive pulse generator under the condition that this range covers at least one octave. From (7) it is clear that (2) applies to all components A_{RFk} since $k \le N$ such that

$$(f_{RFk} \bmod f_S) = (kf_C \bmod f_S) = k(f_C \bmod f_S) < f_{LP} \quad . \quad (8)$$

The final result is an x_{IF} described by N phasors A_{IFk} and corresponding frequencies f_{IFk} as defined by (9) and (10).

$$A_{IFk} = A_{RFk} \quad (9)$$

$$f_{IFk} = k(f_C \bmod f_S) \quad . \quad (10)$$

A practical example illustrates the above. Assume that we have the following constraints on our frequency convertor hardware: f_S ranges from 10 MHz to 20 MHz and f_{LP} equals 4 MHz. These are the actual values for the LSNA prototypes as they were developed by Agilent Technologies. Note that the condition described by (1) is always met. Imagine that one wants to measure a signal with an f_C equal to 1 GHz and with N equal to 3. According to (7) one may choose f_S equal to 19.98 MHz since

$$(1 \text{ GHz mod } 19.98 \text{ MHz}) = 1 \text{ MHz} < 1.333 \text{ MHz} . \quad (11)$$

Applying (2) results in the following set of frequencies at the output of the convertor:
f_{IF1} = 1 MHz, f_{IF2} = 2 MHz and f_{IF3} = 3 MHz.

CONVERSION OF A MODULATED CARRIER WITH HARMONICS

A more complex case occurs when one starts to apply modulation to the carrier and its N harmonics. For LSNA applications the modulation that is applied is usually chosen to be periodic. In this case the signal x_{RF} is a multitone signal that can be described by a set of phasors A_{RFkm} with a set of corresponding frequencies f_{RFkm}, defined as

$$f_{RFkm} = kf_C + mf_M , \quad (12)$$

with f_M being the modulation frequency (the reciprocal of the modulation period). The harmonic index "k" ranges from 0 to N and the so called modulation index "m" ranges from -M to +M. The latter corresponds to a total modulation bandwidth of (2 M f_M). Since each frequency of the set f_{RFkm} is denoted by two indices "k" and "m" the set is called a "dual frequency grid".
The first implementation of acquiring this kind of signals with an LSNA was implemented for

$$(2N + 1)Mf_M < f_{LP} . \quad (13)$$

Since the modulation bandwidth can only be a fraction of f_{LP} this LSNA mode of acquisition is called "narrowband modulation mode". In this mode one chooses f_S in the same way as done in the previous paragraph. The only additional constraint is that one needs to make sure that

$$f_{IF20} - f_{IF10} > 2Mf_M . \quad (14)$$

Note that f_{IFkm} is used to denote the frequency of the frequency converted A_{RFkm}, such that f_{IF10} refers to the frequency converted fundamental carrier and such that f_{IF20} refers to the frequency converted second harmonic. The condition (14) implies that the modulation tones associated with one harmonic do not overlap or coincide with the modulation tones associated with the next harmonic. In other words, the order of the RF frequencies is preserved after frequency conversion. The final result is an x_{IF} described by phasors A_{IFkm} and frequencies f_{IFkm} as defined in (15) and (16).

$$A_{IFkm} = A_{RFkm} \quad (15)$$

$$f_{IFkm} = k(f_C \text{ mod } f_S) + mf_M \quad . \quad (16)$$

A practical example illustrates the above. Consider that the following conditions apply to the signal to be measured: f_C equals 1 GHz, N equals 3, f_M equals 50 kHz and M equals 9. As readily can be verified an f_S equal to 19.98 MHz satisfies both (13) and (14). This results in the following practical values for f_{RFkm} and f_{IFkm}:

$$f_{RFkm} = k \times 1 \text{ GHz} + m \times 50 \text{ kHz} \quad (17)$$

$$f_{IFkm} = k \times 1 \text{ MHz} + m \times 50 \text{ kHz} . \quad (18)$$

The problem with the "narrowband modulation mode" of the sampling frequency convertor is that one needs to make a compromise between N, the number of carrier harmonics to be measured, and M, the number of modulation tones to be measured. This is a natural consequence of (13). An elegant solution to this problem is described in [2]. The corresponding mode is implemented in Agilent's LSNA prototypes and it is called the "broadband modulation mode". Note that the

details of this method are described in [2].
The broadband modulation mode can be applied under the condition that

$$M f_M < f_{LP} . \quad (19)$$

Note that there no longer is a compromise between N and M since N no longer appears in the above equation. In other words, the full IF bandwidth f_{LP} can be used for the modulation tones of the fundamental as well as for all of the harmonics, regardless the number of harmonics. The trick is the choice of f_S. One chooses f_S such that

$$(f_C \bmod f_S) < \frac{f_M}{2N} . \quad (20)$$

This time the condition described in (2) holds for the frequencies f_{RFkm} with m >= 0, while the condition described in (3) holds for the frequencies f_{RFkm} with m < 0. The final result is an x_{IF} described by phasors A_{IFkm} and frequencies f_{IFkm} as defined in (21) and (22):

if m >=0 then $A_{IFkm} = A_{RFkm}$ and $f_{IFkm} = k(f_C \bmod f_S) + mf_M$, (21)

if m < 0 then $A_{IFkm} = \mathrm{conj}(A_{RFkm})$ and $f_{IFkm} = -k(f_C \bmod f_S) - mf_M$. (22)

Consider the following practical example: f_C equals 1 GHz, N equals 3, f_M equals 50 kHz and M equals 60. Note that the total modulation bandwidth equals 6 MHz (= 2 x 60 x 50 kHz). The sampling rate f_C is chosen to be 19.9999 MHz. This results in the following values for f_{IFkm}:

if m >=0 then $A_{IFkm} = A_{RFkm}$ and f_{IFkm} = k x 5 kHz + m x 50 kHz, (23)

if m < 0 then $A_{IFkm} = \mathrm{conj}(A_{RFkm})$ and f_{IFkm} = -k x 5 kHz - m x 50 kHz. (24)

Note the presence of the conjugate in (24).
An original way to graphically illustrate the mapping of f_{RFkm} to f_{IFkm} corresponding to the "narrowband modulation" mode and the "broadband modulation" mode is presented in Fig. 2 to Fig. 8. In these figures each frequency of the set f_{RFkm} or f_{IFkm} is represented by a dot with as x-coordinate the frequency value itself, with as y-coordinate the value of index "m" and with a dot-size proportional to the value of index "k".
Fig. 2 shows the complete set f_{RFkm} for the "narrowband modulation" mode example. Fig. 3 shows a zoom of the same data around a frequency of 2 GHz (the second harmonic of the carrier fundamental). Since the modulation frequency f_M is small compared to f_C all frequencies f_{RFkm} with the same index "k" appear to be lined up vertically in Fig. 2. Fig. 4 represents the corresponding set f_{IFkm} for the "narrowband modulation" mode example. All frequencies f_{IFkm} are uniformly spread from 0 MHz to 3.5 MHz and are ordered the same way as the set f_{RFkm}.
Fig. 5 shows the complete set f_{RFkm} for the "broadband modulation" mode example. Fig. 6 shows a zoom of the same data around a frequency of 2 GHz (the second harmonic of the carrier fundamental). Since the modulation frequency f_M is still small compared to f_C all frequencies f_{RFkm} with the same index "k" also appear to be lined up vertically in Fig. 5. Fig. 7 represents the corresponding set f_{IFkm} for the "broadband modulation" mode example. As can be seen on this figure the order of the set of frequencies f_{IFkm} is completely different from the order of the corresponding set f_{RFkm}. In this mode of operation the sampling frequency convertor behaves more like a frequency scrambler. A zoom around 1 MHz is shown on Fig. 8 and shows the fine structure of the frequency mapping. Each component f_{IFkm} with a positive m has to its left the component $f_{IF(k-1)m}$ and to its right $f_{IF(k+1)m}$, each component f_{IFkm} with a negative m has to its left the component $f_{IF(k+1)m}$ and to its right $f_{IF(k-1)m}$. Note that in "narrowband modulation" mode each component f_{IFkm} has to its left the component $f_{IFk(m-1)}$ and to its right the component $f_{IFk(m+1)}$.

CONVERSION OF EVEN BROADER-BAND MODULATION SIGNALS

The LSNA "broadband modulation" mode enables the measurement of signals with a modulation bandwidth equal to 2 x f_{LP}, which corresponds to 8 MHz for an Agilent LSNA. Recently a method was developed to perform experiments with an even broader modulation bandwidth.

The idea is to first apply a signal x_{RF}, represented by the phasors A_{RFkm} with corresponding frequencies f_{RFkm} as defined by (12), where x_{RF} can be handled by a "narrowband modulation mode" or a "broadband modulation mode". A value is calculated for f_S and at the output of the sampling frequency convertor one gets a signal x_{IF}, represented by the phasors A_{IFkm} with corresponding frequencies f_{IFkm}.

Next one applies a different signal x_{BRF}, represented by the phasors A_{BRFkm} with corresponding frequencies f_{BRFkm} defined as follows:

$$A_{BRFkm} = A_{RFkm} \tag{25}$$

$$f_{BRFkm} = k\, f_C + m\, f_{BM}\ , \tag{26}$$

where $f_{BM} = f_M + q\, f_S$, with q an integer number. Expressed in words, x_{RF} and x_{BRF} only differ by the fact that x_{BRF} has a modulation frequency which is an integer times f_S higher than f_M, the modulation frequency of x_{RF}.

The same value f_S is then used for the sampling rate and at the output of the sampling frequency convertor one gets a signal x_{BIF}, represented by the phasors A_{BIFkm} with corresponding frequencies f_{BIFkm}. Since

$$((k\, f_C + m\, f_{BM}) \bmod f_S) = ((k\, f_C + m\, f_M + m\, q\, f_S) \bmod f_S) = ((k\, f_C + m\, f_M) \bmod f_S), \tag{27}$$

one can conclude that

$$A_{BIFkm} = A_{IFkm} \tag{28}$$

$$f_{BIFkm} = f_{IFkm}\ , \tag{29}$$

$$\text{which actually means that } x_{BIF} = x_{IF}. \tag{30}$$

Expressed in words, the sampling convertor output signal does not change when one increases the modulation frequency by an integer times the sampling rate. This is a very useful result. Consider for example the "narrowband modulation" mode conditions of the previous paragraph where f_C equals 1 GHz, N equals 3, f_M equals 50 kHz and M equals 9. The sampling rate f_C was calculated to be 19.9800 MHz. The frequencies at the output of the sampling frequency convertor will not change if one increases the modulation frequency f_M to e.g. 40.01 MHz (= 50 kHz + 2 x 19.9800 MHz). The corresponding set f_{BRFkm} is then given by

$$f_{RFkm} = k \times 1\text{ GHz} + m \times 40.01\text{ MHz}\ . \tag{31}$$

The set f_{BRFkm} is shown in Fig. 9. Note that all frequencies having the same index "k" no longer appear to line up vertically since the total modulation bandwidth (720.18 MHz) has the same order of magnitude as f_C. The set f_{BIFkm} is equal to the set f_{IFkm} of the "narrowband modulation" mode example shown in Fig. 4.

MEASUREMENTS

The usefulness of the above sampling schemes is illustrated by measurements. These measurements are performed on the Hewlett-Packard LSNA prototype of the Catholic University of Leuven, Belgium. The device-under-test is an on wafer .2µm x 100µm metamorphic HEMT on GaAs provided by IMEC. LSNA two-tone experiments are performed on this component.

For a first experiment the gate of the transistor is excited by a two-tone signal, where one tone has a frequency of 3 GHz minus 100,097.656 Hz and where the other tone has a frequency of 3 GHz plus 100,097.656 Hz. The dual frequency grid which is specified on the LSNA is defined as follows: f_C equals 3 GHz, N equals 6, f_M equals 100,097.656 Hz and M equals 35. The LSNA software calculates a useful f_S equal to 19,999,951.172 Hz, which corresponds to a "broadband modulation" mode. Note that the f_M and f_S have many significant digits, in contradiction to the examples that were given in the previous paragraph. This is caused by constraints related to the LSNA analog-to-digital convertor.

As input signal we consider the incident voltage a_1 at the gate, and as output signal we consider the scattered voltage wave b_2 at the drain of the transistor. Fig. 10 shows the power spectra (dBm) of the fundamental of a_1 and b_2 as measured by the LSNA. In order to avoid cluttering, the frequency axis is given relative to the carrier fundamental (3 GHz). Note that only the components having an index "m" from -10 to +10 are presented. Note the presence of intermodulation products in b_2.

For a second experiment one changes the frequencies of the two-tone experiment by subtracting 7 times f_S from the first tone frequency and by adding 7 times f_S to the second tone frequency. One tone now has a frequency of 3 GHz minus 140,099,755.86 Hz and the other tone has a frequency of 3 GHz plus 140,099,755.86 Hz. Since an integer times f_S has been added to the modulation frequency there is no change in the IF set of frequencies f_{IFkm}. As such the LSNA data acquisition is capable of measuring all spectral components without changing any settings. Fig. 11 shows the measured power spectra (dBm) of the fundamental of a_1 and b_2. The big difference between this figure and Fig. 10 are the MHz units in stead of kHz units on the frequency axis. Please be aware that the example is only given to illustrate the capabilities of the sampling frequency convertor. Significant differences between the power levels shown in Fig. 10 and Fig. 11 are present due to the fact that the standard calibration procedure of the LSNA system was not designed to handle very wide tone spacings and is as such not accurate for this kind of measurements.

The measured spectra of the modulated signals can be represented in the time domain, as shown in [3]. All 6 measured harmonics having each 71 modulation components are taken into account in order to make the time domain representations. Fig. 12 shows the time domain representation of b_2 for the close-to-280 MHz tone-spacing measurement with a time range of about 8 ns. Note that one period of the envelope contains about 21 carrier oscillations. Fig. 13 shows a time domain representation of b_2 for the close-to-200 kHz tone-spacing measurement having the same 8 ns time range. On this time scale no noticeable modulation can be detected. Fig. 14 shows the same b_2 with a much larger time range of 10 µs. This time range corresponds to one period of the modulation envelope and contains no less than 30,000 carrier oscillations. The time domain plots clearly show the orders of magnitude difference in the time scales of the modulation for both two-tone experiments.

CONCLUSIONS

It was explained how a sampling frequency convertor can be used to convert the frequencies of the tones of a broadband periodically modulated carrier and his harmonics into frequencies which are compatible with analog-to-digital convertor technology. Extremely wide modulation bandwidths can be achieved by adding an integer times the sampling frequency to the modulation frequency. The principle was illustrated in the frequency as well as in the time domain by performing a practical LSNA two-tone measurement having a tone-spacing of about 280 MHz.

ACKNOWLEDGEMENTS

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FIGURES

Fig. 1 Schematic of a sampling frequency convertor

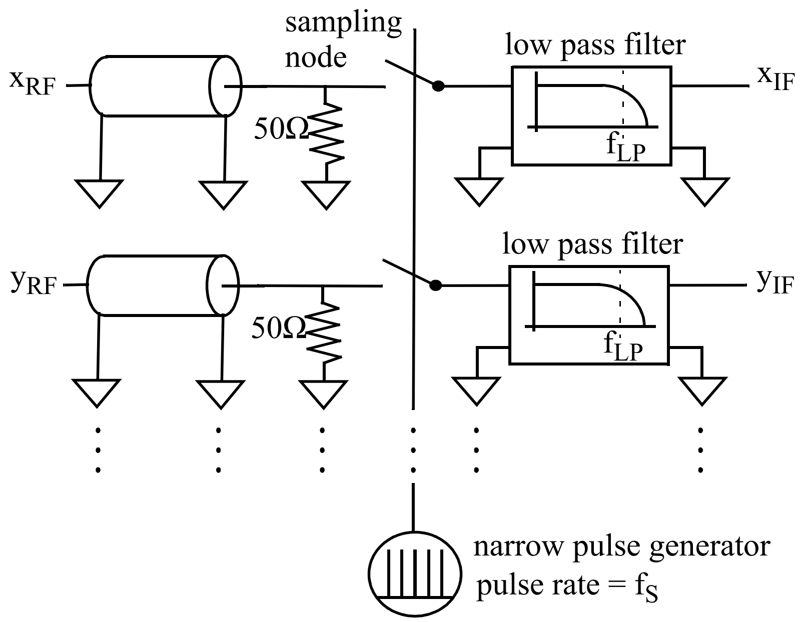

Fig. 2 The set f_{RFkm} for "narrowband modulation" mode

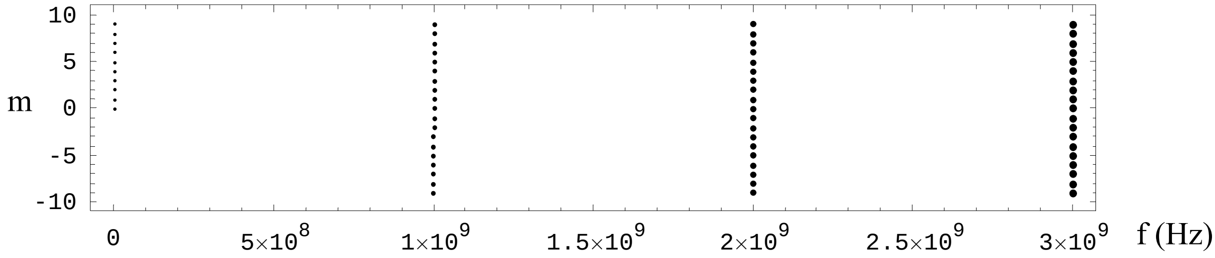

Fig. 3 The set f_{RFkm} for "narrowband modulation" mode (zoom @ 2 GHz)

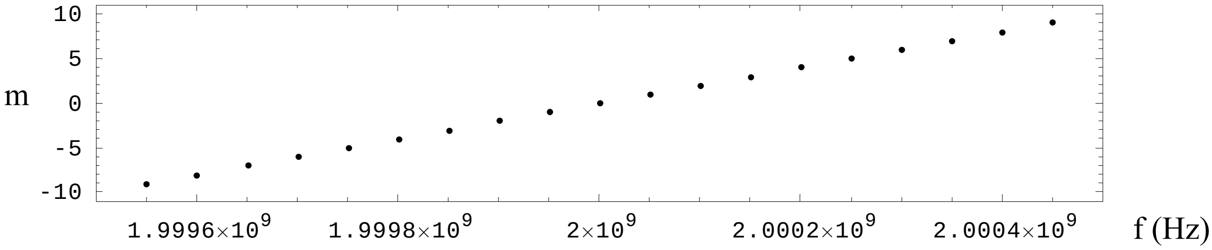

Fig. 4 The set f_{IFkm} for "narrowband modulation" mode

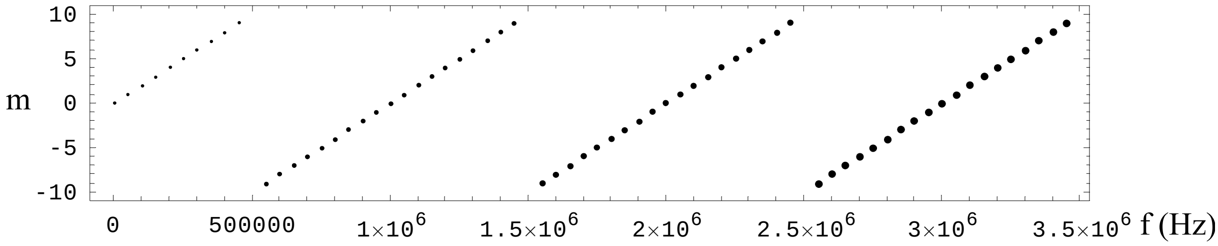

Fig. 5 The set f_{RFkm} for "broadband modulation" mode

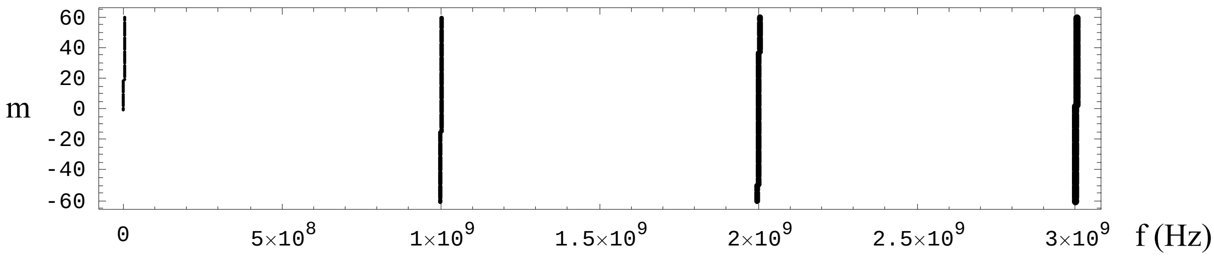

Fig. 6 The set f_{RFkm} for "broadband modulation" mode (zoom @ 2 GHz)

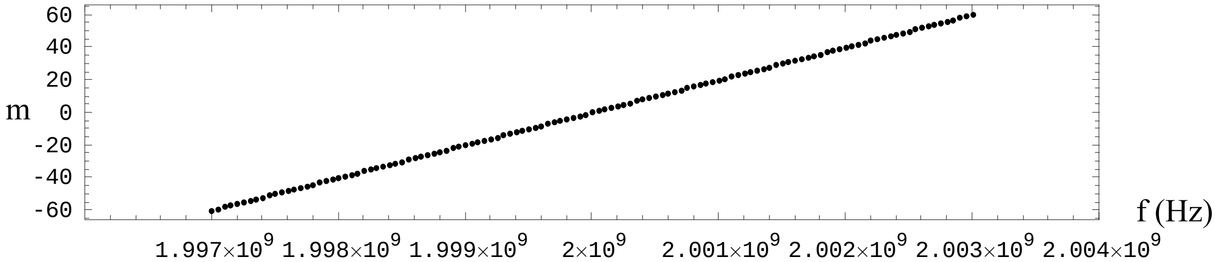

Fig. 7 The set f_{IFkm} for "broadband modulation" mode

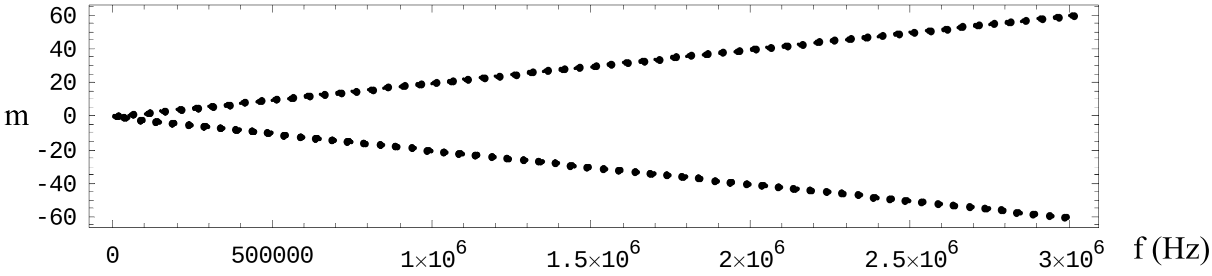

Fig. 8 The set f_{IFkm} for "broadband modulation" mode (zoom @ 1 MHz)

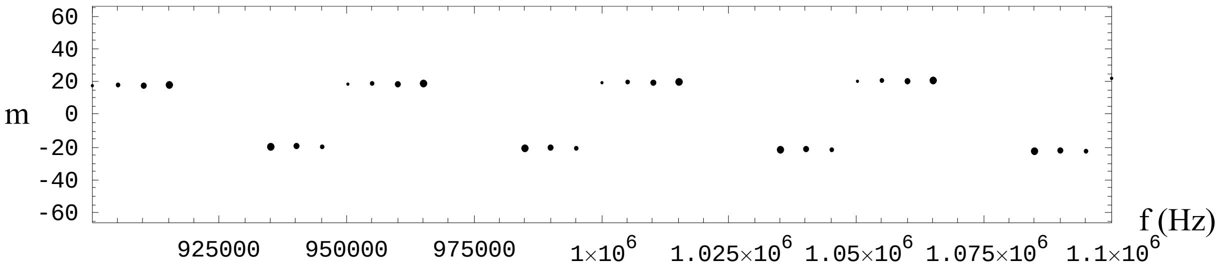

Fig. 9 The set f_{RFkm} for "even-broader band modulation" mode

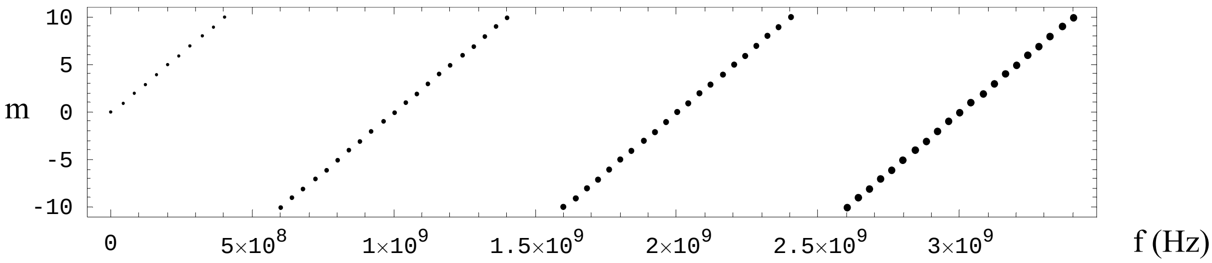

Fig. 10 Two-tone experimental result (tone spacing = 200,195.312 Hz)

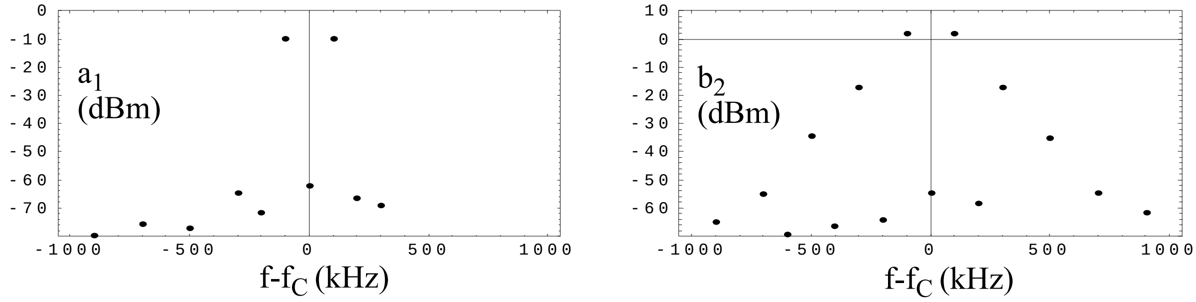

Fig. 11 Two-tone experimental result (tone spacing = 280,199,511.720 Hz)

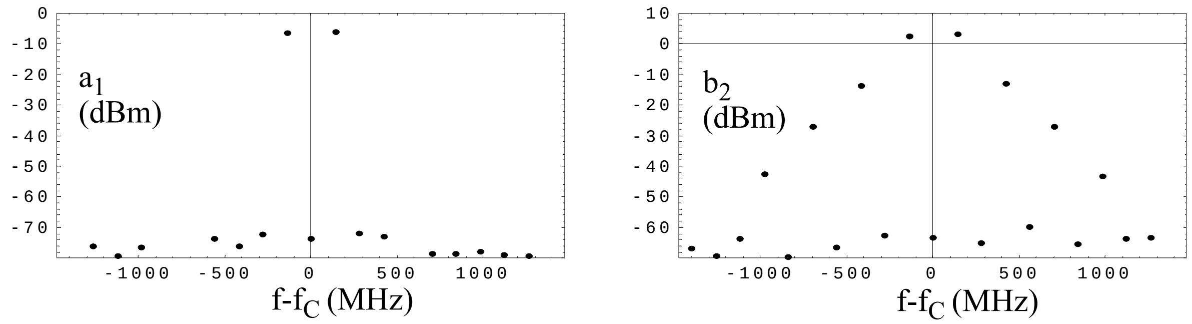

Fig. 12 Time domain waveform b_2 (tone spacing = 280,199,511.720 Hz)

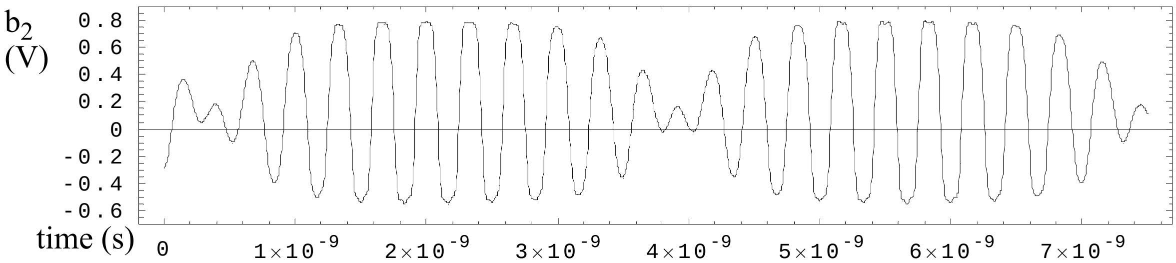

Fig. 13 Time domain waveform b_2 (tone spacing = 200,195.312 Hz)

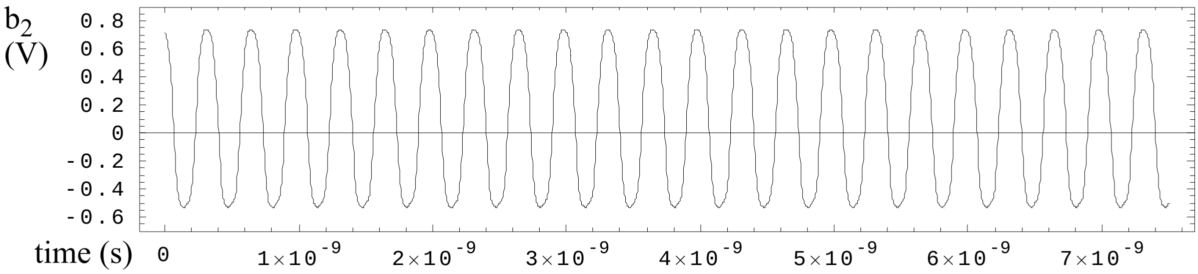

Fig. 14 Time domain waveform b_2 (tone spacing = 200,195.312 Hz)