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Scattering Functions for Nonlinear Behavioral Modeling in the Frequency Domain

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Purpose

- To convince people that

Black Box Modeling

$\neq$

Black Magic Modeling

Scattering functions are a black-box modeling technique for characterizing nonlinear behavior in the frequency domain. I have the impression that many people associate black-box with the practicing of black magic. Today I want to show that this is certainly not true for scattering functions.

Outline

- Introduction
- Theoretical Concepts
- Applications
- Practical Measurements
- Link to Simulators
- Conclusions

Here follows the outline of the presentation.

First we will give a short introduction to the subject

We will then explain the mathematical theory behind scattering functions.

Next is explained how the concept can be used for a variety of applications.

What follows is an explanation on how the scattering functions can be measured in practice.

It is also shown that the scattering functions can easily be linked with harmonic balance simulators.

We will end by drawing conclusions.

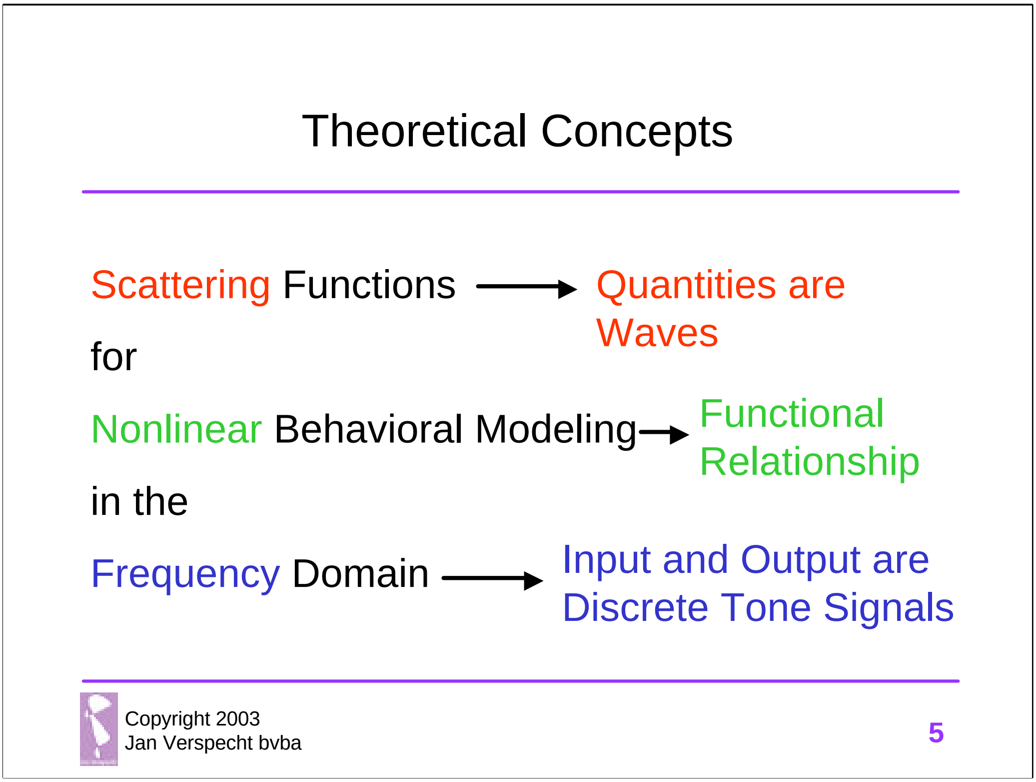
Introduction

- In the late 80’s a need was recognized to “Go beyond S-parameters”
- This implied the development of
  - A mathematical theory to describe certain well defined classes of nonlinear D.U.T. behavior
  - New automated and accurate instrumentation and measurement techniques
- Scattering Functions are one possible solution

In the late 80’s Hewlett-Packard’s network measurement division recognized a growing need to “Go beyond S-parameters”. The goal was the development of a new generation of vector network analyzers with nonlinear measurement capability. HP’s management sponsored the Ph.D. work of Marc Vanden Bossche, which resulted in the foundation of the Network Measurement and Description Group (NMDG), a small R&D group located at the Vrije Universiteit Brussel.

The challenge of “Going beyond S-parameters” implied two things: first of all the development of a mathematical theory to describe well defined classes of nonlinear device-under-test (D.U.T.) behavior and secondly the development of the instrumentation needed for applying the theory.

The R&D work of NMDG resulted in the theory of the scattering functions and in a new instrument called the “Large-Signal Network Analyzer”. The combination of the two provides one possible way of “Going beyond S-parameters”.



We will now explain the theoretical concepts of the scattering functions. They are best introduced by an analysis of the title of this presentation.

The word "scattering" in "scattering functions" refers to the fact that we are working with traveling wave quantities as it is the case with S-parameters. Note that, by convention, we use traveling voltage waves.

"Nonlinear" in "Nonlinear behavioral modeling" implies that, in principle, we will be working with general functional relationships between the wave quantities. This is very different from S-parameters which can only describe a linear relationship.

The words "frequency domain" are present since the scattering functions approach, in the form presented here, work on the assumption of discrete tone signals (multisines) for the incident as well as the scattered waves.

Quantities are Traveling Voltage Waves

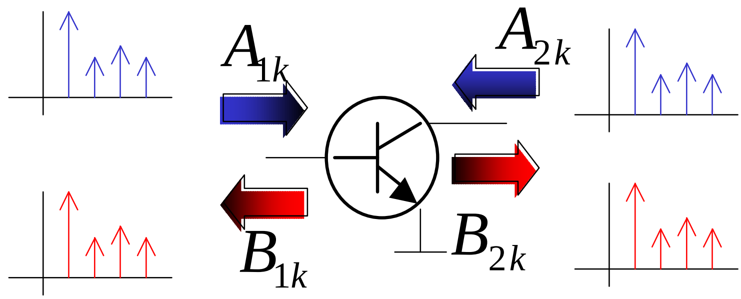
$$\begin{pmatrix} V \\ I \end{pmatrix} \rightarrow \begin{pmatrix} A^{(Z)} \\ B^{(Z)} \end{pmatrix} = \begin{pmatrix} \dfrac{V+ZI}{2} \\ \dfrac{V-ZI}{2} \end{pmatrix}$$

Default value of Z = 50 Ohm (classic S-parameters)

The “traveling voltage waves” are defined as it is the case for classic S-parameters. They are linear combinations of the signal port voltage and current. The default value of the characteristic impedance that is used is 50 Ohms. For certain applications, however, the choice of another value may be more practical. An example are power transistor applications where it may be simpler to use a value which comes close to the output impedance.

Note that the waves are defined based upon a signal port voltage and current and are not associated with a physical wave transmission structure. Therefore the wave quantities are more accurately called “pseudo-waves” (R. Marks and D. Williams, “A general waveguide circuit theory,” J. Res. Natl. Inst. Stand. Technol., vol. 97, pp. 533-562, Sept./Oct. 1992).

Notation - Graphical Illustration



$$B_{1k} = F_{1k}(A_{11}, A_{12}, \ldots, A_{21}, A_{22}, \ldots)$$
$$B_{2k} = F_{2k}(A_{11}, A_{12}, \ldots, A_{21}, A_{22}, \ldots)$$

We will use the following notations.

The incident waves are represented by a capital A and the scattered waves by a capital B. The first subscript refers to the signal port and the second subscript refers to the "harmonic index". Note that an harmonic index of 1 refers to the fundamental component. In general we are looking for the mathematical functions F which correlate the input spectral components A_{1k} and A_{2k} with the output spectral components B_{1k} and B_{2k}.

Phase Normalization

- In order to simplify the mathematics we will use "phase normalized" quantities
- This is especially useful since harmonic frequencies are present
- We will use the dominant large-signal A_{11} component as our phase reference (most useful for many applications)

In order to simplify the mathematics it is important to use "phase normalized" quantities. This is especially useful since harmonic frequencies are present. The "phase normalization" makes it possible to define the phase of an harmonic in a unique way. For the rest of this presentation we will always use A_{11} (the incident fundamental) as our phase reference component. This is typically useful for power transistor and power amplifier applications, where A_{11} is the dominant large-signal input component.

Phase Normalization: Mathematics

- We define a reference phasor: $P = e^{jj(A_{11})}$

- We define phase normalized quantities:

$$A_{mk}^{N} = A_{mk} P^{-k} \qquad B_{mk}^{N} = B_{mk} P^{-k}$$

- Special case: $A_{11}^{N} = \left| A_{11} \right|$

The mathematics of the phase normalization is pretty simple. We define a component P which has a phase equal to the phase of A_{11}, but which has a unity length. The normalized quantities, denoted by the superscript N, are then calculated by multiplying the raw quantities by the reciprocal of P raised to the power k, which corresponds to the harmonic index of that component.

Note that the phase normalized A_{11} is a special case and is equal to a positive real number equal to the amplitude of A_{11}.

Harmonic Superposition Principle

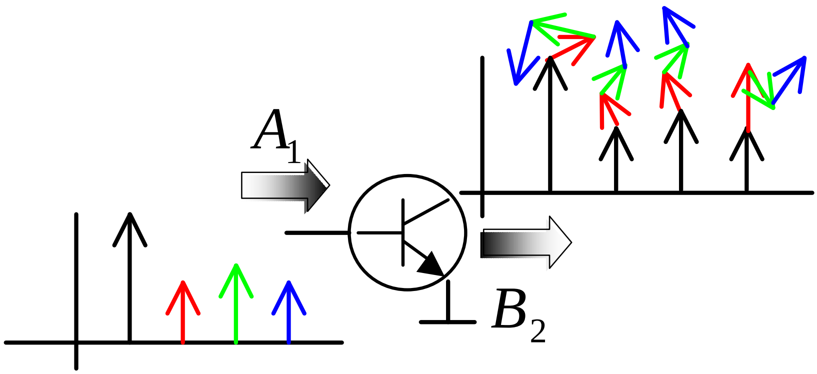
- In general superposition cannot be used to describe the functional relationship between the spectral components

$$F\left(A+A'\right)\neq F(A)+F(A')$$

- The superposition principe can be used for relatively small components, like e.g. harmonics

In general we are not working with linear relationships and the superposition principle is not valid. In many practical cases (e.g. power amplifiers) there is only one dominant large-signal input component present (A_{11}) where as all other input components (the harmonic frequency components) are relatively small. In that case we will be able to use the superposition principle for the relatively small input components. This is called the harmonic superposition principle.

Harmonic Superposition: Illustration



The “harmonic superposition” principle is graphically illustrated above. In order to keep the graph simple we only consider here the presence of an A_1 and B_2 and we neglect the presence of A_2 and B_1. First let us consider the case where only A_{11} is different from zero. The input spectrum A_1 and output spectrum B_2 corresponding to this case are indicated by black arrows. Note the presence of significant harmonic components for B_2. Now leave the A_{11} excitation the same and add a relatively small A_{12} component (second harmonic at the input). This will result in a deviation of the output spectrum B_2, indicated by the red arrows. The same holds of course for a third (green) and a fourth (blue) harmonic. The “harmonic superposition” principle holds when the overall deviation of the output spectrum B_2 is the superposition of all individual deviations. This conjecture was experimentally verified and appeared to be true for all practical power amplifier design cases, whatever the class of the amplifier. The harmonic superposition principle is the key to the definition of the scattering functions.

Basic Mathematical Equation

$$B_{mk}^{N} = \sum_{nh} S_{mknh}(A_{11}^{N})A_{nh}^{N} + \sum_{nh} S'_{mknh}(A_{11}^{N})A_{nh}^{N^{*}}$$

- A_{11} assumed to be the only large-signal component
- Superposition assumed to be valid for other A_{nh}
- The notation A* means the complex conjugate of A
- S and S' are called the scattering functions
- Note that $S'_{mk11} = 0$

The harmonic superposition principle leads to a very simple mathematical formulation which is shown above. The phase normalized B^N waves are the output quantities. They are written as a linear combination of the phase normalized A^N waves, the input quantities, and their conjugates. Note that the resulting expression is linear in all components A_{nh} and their conjugates, except for A_{11} which is assumed to be the only large-signal component for which the superposition principle does not hold. This is an assumption which was experimentally verified and found to be valid in all practical cases of power amplifier design. Since A_{11}^N is a strictly positive real number one can define S'_{mk11} equal to zero without compromising the structure of the equation. The functions S and S', which have both an amplitude and a phase, are the scattering functions. They are a general nonlinear function of A_{11}^N but are independent of the components for which the superposition principle holds. As such they are a natural extension of S-parameters. Note that each S function is denoted by 4 subscripts: the first two to denote the port and harmonic index of the B component, and the latter two to describe the port and harmonic index of the A component.

The scattering functions have some unique features when compared to S-parameters. First of all they relate input and output spectral components which have different frequencies. They can describe e.g. how A_{13}, the third harmonic of the incident wave, will contribute to a change in B_{22}, the second harmonic at port 2. This corresponds to the concept of the "conversion matrix" well known by mixer designers. A characteristic which appears to be counter-intuitive to many people is the existence of S' which is the coefficient associated with the conjugate of the A^N waves. In a nutshell, S' describes the fact that the ratio between the scattered B^N and incident A^N waves depends on the phase of the A components relative to A_{11}.

Applications: Compression and AM-PM conversion

- Only considering B_{21} and A_{11} results in

$$B_{21}^{N} = S_{2111}(A_{11}^{N})A_{11}^{N}$$

- This can be rewritten as

$$S_{2111}(|A_{11}|) = \frac{B_{21}}{A_{11}}$$

- $S_{2111}(|A_{11}|)$ represents the compression and AM-PM conversion characteristic

The scattering functions formalism covers many applications. The S_{2111} function can be interpreted as the compression and AM-PM conversion characteristic of a device. Note that defining the resulting compression and AM-PM conversion characteristic this way implicitly assumes that it is independent from harmonic and fundamental incident waves. Classic compression and AM-PM characteristics are usually measured on systems having imperfect matching characteristics. As a result classic measurements of these characteristics differ from measurement system to measurement system because of the presence of different matching characteristics. The S_{2111} numbers returned by a scattering functions measurement instrument compensates for the non-ideal instrument port match. This is actually similar to what happens with a vector network analyzer. Although the port match of two VNA's may significantly differ, the S-parameters returned are not affected. As such the measured scattering functions are true device characteristics, not disturbed by instrument imperfections.

Large-Signal Input Match

- Only considering B_{11} and A_{11} results in

$$B_{11}^{N} = S_{1111}(A_{11}^{N})A_{11}^{N}$$

- This can be rewritten as

$$S_{1111}(|A_{11}|) = \frac{B_{11}}{A_{11}}$$

- $S_{1111}(|A_{11}|)$ represents the large-signal input reflection coefficient

In a similar way the S_{1111} function can be interpreted as the large-signal input reflection coefficient.

Hot S_{22}

- Considering B_{21}, A_{21} and A_{11} results in

$$B_{21}^{N} = S_{2111}(A_{11}^{N})A_{11}^{N} + S_{2121}(A_{11}^{N})A_{21}^{N} + S'_{2121}(A_{11}^{N})A_{21}^{N^{*}}$$

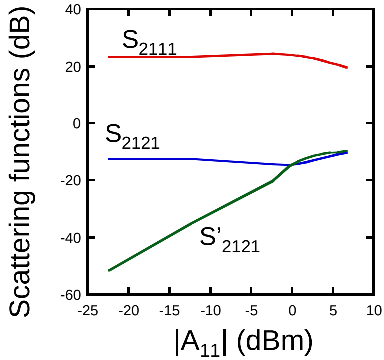
- Multiplying both sides with P results in

$$B_{21} = S_{2111}(\left|A_{11}\right|)A_{11} + S_{2121}(\left|A_{11}\right|)A_{21} + S'_{2121}(\left|A_{11}^{N}\right|)P^{2}A_{21}^{*}$$

- The combinaton of S_{2121} and S'_{2121} are a scientifically sound format for "Hot S_{22}"

Looking at S_{2121} and S'_{2121} results in an original and scientifically sound definition of "Hot S_{22}". It is important to understand that "Hot S_{22}" behavior can only be tackled in a scientifically sound way by using the combination of S_{2121} and S'_{2121}. By our knowledge this is an original result. Classic "Hot S_{22}" approaches completely ignore the existence of the S'_{2121} component. When written in quantities which are not phase normalized we see the appearance of P^2.

Measurement Example



- Note that the amplitude of S'_{2121} becomes arbitrary small for $|A_{11}|$ going to zero

Above we show a practical measurement of the amplitude of S_{2111}, S_{2121} and S'_{2121} as a function of the amplitude of A_{11}.

We clearly see the compression characteristic of S_{2111}. Especially interesting is the analysis of the behavior of the scattering functions for small amplitudes of A_{11}. For small A_{11} amplitudes S_{2111} and S_{2121} are constants, equal to the classic S-parameters s_{21} and s_{22}. S'_{2121} on the contrary becomes arbitrarily small for small amplitudes of A_{11}. This illustrates the fact that the component S'_{2121} is only visible under large-signal (nonlinear) operating conditions. Note that the amplitude of S'_{2121} becomes significant at relatively low levels of compression. As such problems can be expected with classic "Hot S_{22}" approaches since they completely neglect the existence of this component.

Harmonic Distortion Analysis

- Only considering A_{11} and B_{2k} one can calculate the harmonic distortion as a function of $|A_{11}|$

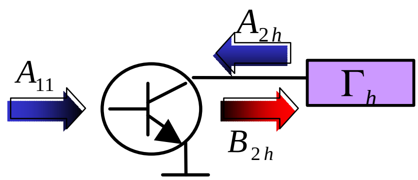
$$B_{21} = S_{2111}(|A_{11}|)A_{11}$$

$$B_{22} = S_{2211}(|A_{11}|)A_{11}P$$

$$B_{23} = S_{2311}(|A_{11}|)A_{11}P^2$$

The scattering functions contain all the information that is needed to calculate the phase and amplitude of the harmonics as a function of the input amplitude. They can as such be used for harmonic distortion analysis.

Harmonic Loadpull Behavior



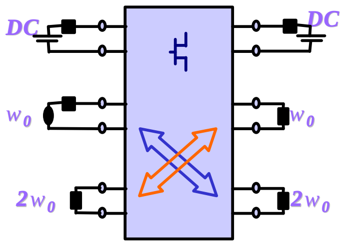
$$B_{2k}^N = \sum_h S_{2k2h}(A_{11}^N)A_{2h}^N + \sum_h S'_{2k2h}(A_{11}^N){A_{2h}^N}^*$$

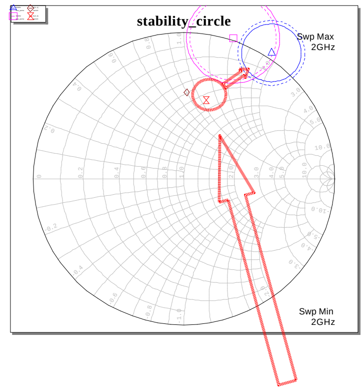
$$A_{2h}^N = \Gamma_h B_{2h}^N$$

- Solve the set of equations
  (linear in the real and imaginary parts of A_{2h} and B_{2h})

An important application is the prediction of fundamental and harmonic loadpull behavior. In this case we want to predict the B_{2h} waves (particularly B_{21}) as a function of the matching conditions at the output, both for the fundamental and the harmonics. This can be achieved by solving the above represented set of equations. The first set of equations represent the scattering functions, the second set mathematically represents the matching conditions. Note that the set of equations is linear when one considers the real and imaginary parts of A_{2h} as separate variables and is as such easy to solve.

New Stability Circles for Multiplier Design





Research performed by
Prof. Giorgio Leuzzi
(Universita dell'Aquila, Italy)

Stability is not ensured

A recent application is the definition of large-signal stability circles. They are especially useful for the stability analysis encountered with the design of frequency multipliers. This research is performed by Prof. Giorgio Leuzzi of the Universita dell'Aquila (Italy). In this research work it is shown that the scattering functions are very closely related to the Jabobian as it is used in harmonic balance simulators.

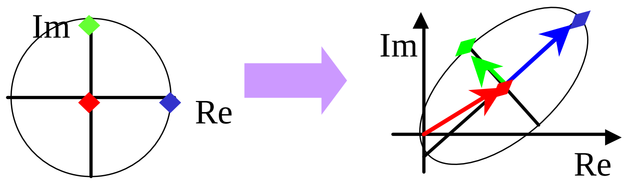
Practical Measurement: Experiment Design Concept

- Simple example: S_{2111}, S_{2121} and S'_{2121}

$$B_{21}^N = \underline{S_{2111}(A_{11}^N)A_{11}^N} + \underline{S_{2121}(A_{11}^N)A_{21}^N + S'_{2121}(A_{11}^N){A_{21}^N}^*}$$

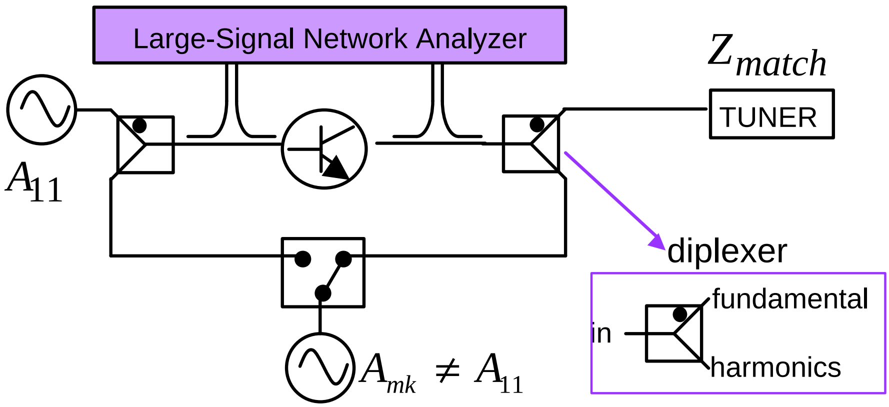
- Perform 3 independent experiments

Input A_{21} Output B_{21}



The experiment design concept to extract the actual values of the scattering functions is simple. Assume that we want to know S_{2111}, S_{2121} and S'_{2121} for a particular amplitude value of A_{11}. We will apply the particular A_{11} and keep it constant during the rest of the experiment. First we will not apply any other incident wave besides A_{11}(this experiment is represented by the red square). This results in the knowledge of S_{2111}. Next we will perform two more independent experiments, one with applying an A_{21} with a zero phase and one with a 90 degree phase (corresponding to the blue and respectively green square). Having these two additional measurements we have sufficient information to calculate S_{2121} and S'_{2121}.

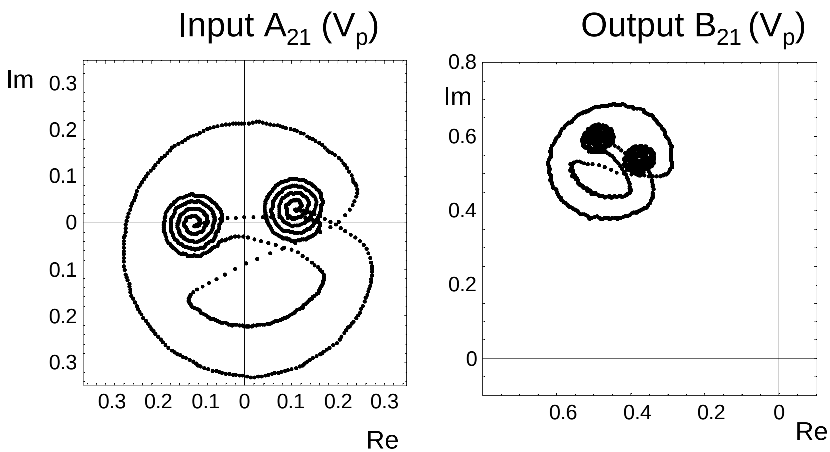
Typical Measurement Setup



Agilent Technologies, Inc. - Patent Pending

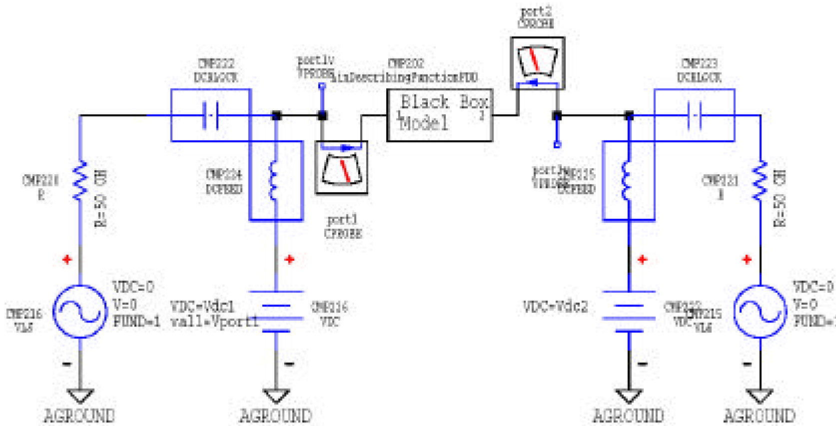
A typical measurement setup is shown above. A Large-Signal Network Analyzer measures all relevant A_{mk} and B_{mk} components. One synthesizer (typically used in combination with a power amplifier) is used for the generation of the A_{11} component. A second synthesizer, combined with a switch, is used for the generation of the harmonic “small signal” components A_{mk}. The fundamental component A_{21} is often generated by using a tuner. The diplexers are there to decouple the fundamental and the harmonic behavior. Note that there is a patent pending with Agilent Technologies related to this technique.

Measurement Example



Although 3 measurements are theoretically sufficient to extract the scattering functions one usually performs many more measurements in combination with a linear regression technique. A practical example of such a set of experiments is shown above ☺. The presence of redundancy in the measurement set offers many possibilities in the framework of system identification (gathering information on noise errors and residual model errors).

Link to Harmonic Balance Simulators



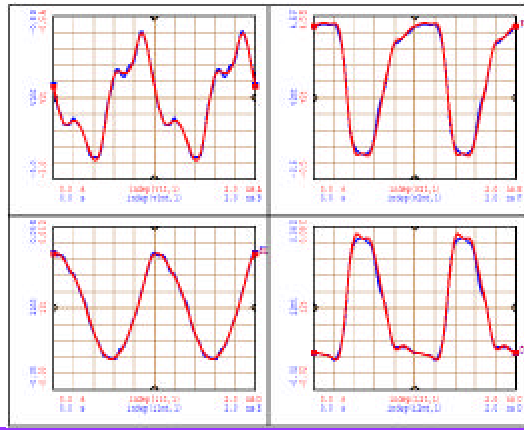
The scattering functions can easily be linked to harmonic balance simulators. In fact their mathematical structure fits the harmonic balance simulator like a glove. This results in memory efficient and fast simulations. Model accuracy is ensured by the fact that the scattering functions are directly derived from measurements. The accuracy statement holds as far as the device-under-test is stimulated under conditions for which the assumed harmonic superposition principle holds.

Simulated Model versus Measurements

Powertransistor Waveforms

Gate Voltage

Drain Voltage



Gate Current

Drain Current

The above graph represents a comparison between the measured and modelled (by means of the scattering functions) time domain current and voltage waveforms at the terminals of a power transistor under loadpull conditions. Note that the loadpull condition was arbitrarily chosen and was not part of the experiment set to extract the scattering functions. As one can see the correspondance is striking and should clearly be sufficient for practical power amplifier design. Note that the modelled waveforms were calculated by putting the scattering functions in an harmonic balance simulator as shown on the previous slide.

Conclusions

- scattering functions are a Black Box modeling approach without Black Magic
- They can accurately describe many classic concepts like e.g. AM-PM, compression, harmonic distortion, Hot S_{22}, loadpull,…
- They can automatically be measured using a Large-Signal Network Analyzer
- They are compatible with harmonic balance simulators

This brings us to the conclusions of this presentation.

I hope that I was able to convince you that the presented scattering functions are a black box modeling approach without black magic.

The scattering functions can accurately describe many classic concepts like e.g. AM-PM conversion, compression, harmonic distortion, Hot S_{22}, loadpull behavior,…

They can automatically be measured by using a Large-Signal Network Analyzer with the addition of the appropriate experiment generation hardware.

They are fully compatible with harmonic balance simulators.

I truly believe that the scattering functions have a lot of potential and are a promising way to take us all “Beyond S-parameters”.