Broad-Band Poly-Harmonic Distortion (PHD) Behavioral Models From Fast Automated Simulations and Large-Signal Vectorial Network Measurements

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design. This refers to the set of experiments (stimuli) required to elicit the system response sufficiently to identify the model parameters. The third is the model generation procedure. This is the algorithm used to determine the model parameters from the data obtained from the experiments.

Requirements for a robust behavioral modeling process include the ability to characterize, quickly, the component or circuit-level model, and construct the behavioral model in a repeatable, procedural, and automated way. The excitations should be as few in number as possible for the particular model class. Ideally, each experiment can be used to identify a particular model parameter uniquely and independently. In an optimal design, each additional experiment provides totally new, "orthogonal" information [4].

In [5], a black-box frequency-domain behavioral model, generalized from the work of [6], was identified from real automated measurements on a wide-band microwave IC amplifier using a VNNA [7]. This measurement-based behavioral model was experimentally demonstrated to be valid for small and large amplitude drive signals, correctly predict even and odd harmonics, and simulate accurately even into impedances different from the 50- environment in which the data was measured. One limitation of the usefulness of the model is the limited dynamic range of the VNNA instrument, which can be estimated from the data in [5]. Nevertheless, the results of [5] demonstrate that the behavioral model, together with the automated VNNA measurements to identify it, provide a general, practical, and useful tool. Moreover, recent large-signal hardware developments [8] significantly demonstrate improved dynamic range that will only increase the general utility of the approach. Another limitation of [5] is the suboptimal nature of the experiment design and model identification algorithm. That is the subject of this study.

In this paper, we present a superior experiment design approach and an improved algorithm for identifying, from this different set of data, the behavioral model discussed in [5]. In fact, the approach is both orthogonal and optimal in the sense it uses the minimum number of independent measurements. We apply the new approaches to generate accurate behavioral models from detailed circuit-level models of real microwave ICs using the nonlinear simulator as a virtual instrument. New results, including the prediction of ACPR and I–Q constellation diagrams by the behavioral model are presented and validated. In combination with [5], this study completes the "closing of the loop" to include both simulation- and measurement-based approaches to generating the same frequency-domain nonlinear behavioral model.

In Section II, we briefly review the poly-harmonic distortion (PHD) behavioral model. In Section III, we describe the new experiment design and model generation algorithms. In Section IV, we compare the approach to other work in the literature. In Section V, we present new results validating the PHD model against the circuit model from which it was derived.

II. PHD Model Formulation

The target behavioral model for this study was presented in [5], which generalized the work first presented in [6] and summarized in [2, Ch. 5]. It is a "black-box" behavioral model requiring no *a priori* knowledge of the device physics or circuit configuration of the nonlinear component. The model theory derives from a multiharmonic linearization around a periodic steady state determined by a large-amplitude single input tone. For this reason, we refer to the model as the PHD model. The assumption is that the system to be modeled can depend in a strongly nonlinear way on its large-signal drive, but nevertheless responds linearly to additional signal components at the harmonic frequencies considered as "small" perturbations around the time-varying system state. This is referred to as the "harmonic superposition" principle [6]. The harmonic superposition principle has been shown in [2, Ch. 5], [5], and [6] to be an approximation well satisfied by power amplifiers of several different classes and for applications where the functional block is inserted into impedance environments mismatched somewhat from 50 at both the fundamental and harmonics. In real applications, for example, these harmonic terms can result from nonlinearities created from previous amplification stages or reflections from nonlinear devices at the next input stage of a multistage amplifier. The broad-band nature of the model is essential for modeling the frequency dependences of the nonlinear responses of such microwave ICs as multiple-octave traveling-wave amplifiers and other components useful in instrument applications.

The model is defined by (1) and (2) in the frequency domain relating complex transmitted and scattered waves at each port and harmonic index to a linear combination of terms in the incident waves and their complex conjugates independently at each port at each harmonic. The fact that the complex conjugate terms appear is a necessary consequence of the nonanalyticity of the Jacobian, which represents the linearization around the time-varying operating point established by the single large-amplitude tone in the absence of perturbation. An alternative explanation follows from the mixer analysis of Section III. The sums in (1) are over all port indexes, and harmonic indices (DC is excluded in the cases presented here so the sum over starts at the fundamental. In general, this method can easily be extended to include the dc term, in which case, the sum starts from index 0.)

(1)

(2)

In (1), is a pure phase that, along with the magnitude-only dependence on of the and functions, is a necessary consequence of the assumed time invariance of the underlying system. A redundancy, introduced by summing over the fundamental components in addition to the harmonics in (1), requires the imposition of the additional constraints given by (2). For all but one of the applications demonstrated in Section VI, we consider a two-port amplifier model with five harmonics. The final result was obtained considering only three harmonics.

III. Excitation Design

In [5], the excitation design for the PHD model was based on perturbing the nonlinear component under a large-signal drive by applying several small tones one at a time at each port and at each harmonic of the fundamental. This was done for each harmonic up to the maximum number needed for the model (or, for the measurement-based case, the limitation of the instrument bandwidth). The structure of model (1) and (2) is such that, in principle, the and coefficients at each harmonic can be extracted directly from three measurements. These measurements are: 1) the responses at each port and at each harmonic frequency to the large tone without perturbation; 2) the responses to the simultaneous excitation of the large tone and a small-signal perturbation tone; and 3) the responses to a simultaneous excitation of the large tone and a small-signal perturbation tone at the same frequency, but different phase compared to the small tone of 2). The component of the-wave at each port and at each harmonic has contributions from both and ; the two relative phases per frequency per port for the small tones were proposed in order to provide two independent data for from which to determine the two model coefficients (and) for a given harmonic frequency component of the response.

The improved experiment design is based on considering model (1) and (2) as the limiting case of a more general time-varying nonlinear system perturbed by an arbitrary small tone. Here the restriction that the frequency of the perturbation tone is exactly at a harmonic of the fundamental is relaxed. Such a system can be analyzed as a mixer. Moreover, if the perturbing tone is sufficiently small, the analysis can be considered to be that of a "small-signal mixer" (SM).

The derivation is outlined for a single port. The extension to multiple ports is obvious. We start in the time domain by representing the output wave as a nonlinear function of the input wave according to (3) as follows:

(3)

These are real signals and, in (3), the nonlinearity is algebraic (this restriction is not necessary, but facilitates a simpler way to the result). We now consider the input signal class to be the sum of a single large tone at frequency and a small-signal at frequency as follows:

(4)

We assume the perturbation is small, and expand (4) in a Taylor series and keep terms only up through first order as follows:

(5)

Identifying , the linear response is given by (6) as follows:

(6)

Since is periodic, we can expand the first term on the right-hand side of the equal sign of (6), the conductance nonlinearity, in a Fourier series in as follows:

(7)

The perturbation tone is represented in the frequency domain as follows:

(8)

Here, and is a small (in magnitude) complex number.

Note that we are dealing with two periodic signals with unrelated fundamental periods (7) corresponding to the system response to a large tone at, and (8), the small tone at . Multiplying out the factors in (6) using (7) and (8) results in the following expression for :

(9)

For future reference, we designate the third through sixth terms of (9) as (a)–(d), respectively.

We now consider the special case where the frequency of the small tone is nearly a harmonic (integer multiple) of the fundamental of the large tone, i.e., . Here, is a positive infinitesimal. The frequency offset will allow us to refer to the stimulus at frequency as the upper sideband stimulus at frequency .

The objective is to pick out the complex spectral components of the response in the frequency domain at the harmonic frequencies for nonnegative integers . We can break up the contributions into terms proportional to and separately. For simplicity, we assume there are no dc components in the following. Looking at terms (a) and (b) in (9), we find that the contributions that are proportional to are for or for . From terms (c) and (d) in (9), we obtain the terms proportional to . The results are for and for .

Thus, we can write the linearized response at the th harmonic in response to the perturbation at the th harmonic as

(10)

with the coefficients given in terms of the harmonic series for the conductance as described above. This allows the behavior of the and coefficient functions to be related to the Volterra representation of the original nonlinearity.

If we compare (10) (note we omitted the port indexes here) to (1), we can see that the coefficients of the PHD model can be explicitly calculated in terms of the Fourier series of the system conductance nonlinearity of (7) in the limit as goes to zero.

TABLE I
OUTPUT OF SM SIMULATION F = 3 GHz, M = 2, AND ORDER = 8

Keeping track of the term, we can also see that the coefficients are the responses at the upper sideband and that the coefficients, with the same indices, are the lower sideband responses. This is the direct way to identify the PHD model from this SM analysis.

From basic mixer theory, if a signal consisting of the sum of two tones at (angular) frequencies and , respectively, are put through a nonlinear device, the discrete frequencies of the response fall at frequencies satisfying

(11)

for and provided (to keep from double counting). The integers and correspond to the order of the mixing terms. If we further assume that one tone is always small compared to the other, we can simplify (11) by assuming all terms beyond the first order of the small tone can be neglected. This is equivalent to restricting .

We now set . For , we get the degenerate case of the harmonically related experiment of the design approach of [5]. This corresponds to a small tone at the th harmonic of the fundamental. Considering as a positive infinitesimal allows us to keep separate track of the two different terms that contribute to the same frequencies in the output spectrum due to different origins.

We consider the example for which GHz, (small tone at 6 GHz), and order . Here "order" is the order of the HB analysis part of the SM analysis used. The spectral response, linear in the perturbation signal, can, therefore, be represented as in Table I. This represents the difference between the full output spectrum of the system with one large and one small tone, and the output of the system with only the single large tone (no small tone added). The second column is the order of the large tone contributing to the frequency component specified in that row. A negative sign means the negative frequency component. The third column indicates the order of the small tone (recall only terms for are considered). The fourth column indicates whether the contribution to this frequency is at the lower or upper sideband (by keeping track of). We could also distinguish the sidebands simply by checking the value of . There is only one contribution at frequency ; this is an upper sideband. There are two contributions to the next group of frequencies, from 3 to 18 GHz, in the same alternating order of lower and upper sidebands. This follows from the two different contributions of orders of the large and small tones that can combine to give terms at each of these frequencies. For example, at 9 GHz, the fifth-order contribution from the large tone at 3 GHz combines with the negative frequency component of the small tone at 6 GHz to give a tone at GHz GHz GHz. The upper sideband comes from a combination of a first-order term in the large tone with the positive term from the small tone because GHz GHz GHz. There are no other combinations possible to end up at 9 GHz. Eventually, at 21 GHz, there are only upper sidebands. This is because for a lower sideband to exist, it must correspond to the solution of (12) as follows:

GHz GHz GHz (12)

The solution to (12) is , which is beyond the order value, and thus it is not calculated. This condition persists for the rest of the frequencies. There is a table like this for each value of .

In a simulation, using SM analysis, to be described, we can set . In a real measurement, however, must be kept small, but nonzero, typically approximately 1 kHz. In this case, there are always both upper and lower sidebands at each frequency provided their magnitude is large enough to measure and if the frequency offset is not too small to resolve the two sidebands. Therefore, in a real measurement, there would be second rows in the table just below 21, 24, 27, and 30 GHz (for this example) corresponding to the lower sidebands.

Through these calculations we determine the coefficients from the upper sideband responses and the coefficients from the lower sideband responses. This demonstrates that we only need a *single* upper sideband (small) signal excitation at each port at each harmonic from which to extract both and coefficients corresponding to upper and lower sidebands, respectively. We do not need (at least) two small tones of different relative phases as required by the method of [5].

For the simulation-based approach, the Agilent Advanced Design System (ADS) SM analysis is used as the excitation. A key advantage of this excitation is that the simulation is much faster than a two-tone HB analysis since the only HB analysis done in the former is that for the single large tone. The linearization of the system is done automatically using the Jacobian information already computed by the simulator for the one-tone HB analysis. Another advantage is that the SM analysis results in exactly (to numerical precision) the